Time varying mean extraction for stationary and nonstationary winds

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A R T I C L E   I N F O

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A B S T R A C T

This paper discusses different strategies for the extraction of the time-varying mean from wind speed time histories. Due to the advantage of allowing analytical evaluations, the attention is focused on kernel regression techniques, considering different weighting functions, namely a constant, a Gaussian and a cardinal sine weighting function. The problem is firstly treated analytically, and the frequency-domain properties of the filter associated to different kinds of weighting functions in the definition of the slowly varying mean through kernel regression are analysed. Then, different weighting functions are adopted for the analysis of digitally-simulated stationary wind speed time histories and for the time histories of thunderstorm outflows recorded by a tri-axial anemometer. The consequences of the adoption of different weighting functions on the harmonic content and statistical properties of turbulence are studied. The same features are found also for thunderstorm outflow records.

1. Introduction

Extra-tropical cyclones at the synoptic scale present a harmonic content characterized by two peaks (macro- and micro-meteorological), and a spectral gap corresponding to time intervals between 10 min and 1 h (Van der Hoven, 1957). Thus, in wind engineering, considering time intervals in the range between 10 min and 1 h, the wind speed is commonly decomposed as the sum of a constant mean component, related to the macro-meteorological peak, and a time-varying turbulent fluctuation, corresponding to the micro-meteorological peak, modelled as a Gaussian stationary random process (Solari, 2019). Based on this decomposition, the wind-excited dynamic response of structures is decomposed as the sum of a mean component, estimated as the static response to the man wind actions, and a Gaussian stationary random component, i.e. the dynamic response to turbulent fluctuations.

Extreme winds such as tornadoes, thunderstorm downbursts and hurricanes/typhoons are characterized by a probability distribution different from the one characterizing synoptic winds (Zhang et al., 2018; Lombardo and Zickar, 2019), a more or less rapid time-varying mean and stationary/non stationary fluctuations (e.g. Chen and Xu, 2004; Xu and Chen, 2004; Chen and Letchford, 2005), thus the classical decomposition into a constant mean wind speed and a stationary turbulence cannot be applied. However, also for these phenomena, the decomposition into a slowly-varying mean and a fluctuating component is usually adopted.

A number of techniques have been used to derive the time-varying mean for non-stationary winds (Choi and Hidayat, 2002; Lombardo et al., 2014; Chen and Letchford, 2005; McCullough et al., 2014; Huang et al., 2015; Solari et al., 2015; Su et al., 2015; Zhang et al., 2018, 2019; Junayed et al., 2019). They include the moving average technique, adopting a suitable time interval (Choi and Hidayat, 2002; Lombardo et al., 2014; Solari et al., 2015; Zhang et al., 2018, 2019; Junayed et al., 2019) and a broad class of advanced procedures such as the discrete wavelet transform, empirical mode decomposition and variable averaging interval schemes (Chen and Letchford, 2005; McCullough et al., 2014; Su et al., 2015; Huang et al., 2015).

These latter techniques are generally considered as more refined, although they are often based on a subjective choice of the parameters (e.g. the wavelet type and order and the number of decomposition levels to be taken into account in wavelet transform, and the number of lower level intrinsic mode functions to be subtracted in empirical mode decomposition). Under this point of view the moving average can be considered as the simplest and most straightforward procedure, only requiring the definition of the averaging time interval (usually between 10 s and 40 s). However, the moving average technique can also be adopted by selecting suitable weighting functions (Su et al., 2015): these procedures have some noticeable advantages, first of all they allow an analytical interpretation of the filtering effect associated with the moving average wind speed extraction.

Once the slowly-varying mean wind speed has been extracted, the...
residual fluctuation is usually modelled as a modulated stationary random process with zero mean and unit variance, referred to as the reduced turbulent fluctuation. The modulating function was initially identified with a fraction of the slowly-varying mean (Chen and Letchford, 2004; Chay et al., 2006), later with the slowly varying standard deviation (Chen and Letchford, 2007; Holmes et al., 2008; Solari et al., 2015a; Zhang et al., 2018, 2019). The studies carried out by Zhang et al. (2018b, 2019) showed that the reduced turbulence fluctuation of both thunderstorm outflows and synoptic winds tend to be very similar provided they are extracted using the same procedure.

Despite a lot of research has been carried out in this field, several aspects seem to be not enough investigated and clarified. In particular, the moving average technique has been extensively used without exploring the advantages and disadvantages provided by different weighting functions. The extraction of the slowly-varying mean wind speed and the turbulence modulation have been often carried out sequentially without discussing the different properties and implications of these two operations, especially with respect to the accurate assessment of the dynamic response of structures. Research has demonstrated that the turbulence intensity for thunderstorm events is generally smaller than the classical one for synoptic winds, due to the removal of the low frequency harmonic turbulence components related to the moving average operation (Zhang et al., 2018b, 2019). However, a rational discussion about this reduction is not yet available in the literature. At present, no study has been carried out concerning the cross-power spectral density function (cpsdf) and/or the correlation between the slowly-varying mean and the fluctuating component. This information may be essential in the analysis of the dynamic response of structures based on the decomposition of the wind velocity into a slowly-varying mean and a fluctuating component (Denoël, 2009). Furthermore, different methods have been used in the literature for the estimation of the dynamic response of structures to thunderstorms (Chen, 2008; Kwon and Kareem, 2009, 2019; Solari et al., 2015b; Solari, 2016; Solari and De Gaetano, 2018; Le and Caracoglia, 2018; Hangan et al., 2019): the effect of the adoption of different techniques for the separation of the mean and fluctuating part of the wind velocity on the dynamic response estimate has not been investigated.

This paper is part of the activities carried out for the Project THUNDER - Detection, simulation, modelling and loading of thunderstorm outflows to design wind-safer and cost-efficient structures - supported by an Advanced Grant (AdG) 2016 (Solari et al., 2020). Its objective is to discuss the possible strategies for the slowly-varying mean wind speed extraction by means of weighted moving average procedures, inspecting and clarifying the consequences of different extraction procedures and different averaging times on both the slowly-varying mean wind speed and the residual fluctuation, also with respect to the accurate assessment of the dynamic response of structures. In order to provide a theoretical insight into the problem, such consequences are first studied analytically in general terms (Section 2), then with regard to both synoptic and thunderstorm velocity records. Dealing with synoptic winds (Section 3), analyses are first carried out analytically starting from power spectral density function (psdf) models for the three turbulence components (Section 3.1), then numerically, on statistically representative samples of simulated time histories (Section 3.2). The analysis of stationary events allows to formulate useful observations propaedeutic to the statistical analysis of a selected set of thunderstorm events measured by one anemometer (Section 4). In particular, some consequences of the moving average extraction rigorously demonstrated for synoptic stationary events provide interesting interpretations discussed with regard to thunderstorm records. Finally, Section 5 summarizes the most relevant conclusions and discusses the main prospects for further analyses.

2. Analytical formulation

Let us consider a function of the time \( t \), \( x(t) \), in the interval of time \([0, T_f]\). The mean value can be defined through different criteria according to the analysed phenomenon. It can be defined as a constant value estimated through the mean of the signal along the whole time history \( T_f \) or it can be estimated as the moving average along a period \( T \) shorter than the total duration \( T_f \) or through different techniques such as Kernel Regression, Wavelets, Empirical Mode Decomposition. In this paper, kernel regression technique is considered due to the possibility of an analytical formulation, the avoidance of several subjective evaluations and the clear interpretation of its results. In this framework, the moving average technique can be seen as a particular kernel regression, carried out adopting a constant weighting function.

In the following, Section 2.1 provides the analytical formulation for the mean extraction through the kernel regression whereas Section 2.2 is devoted to the definition of the reduced fluctuation.

2.1. Mean extraction

Adopting the kernel regression technique, the mean \( \bar{x}_m(t) \) (slowly-varying component) can be expressed through the following convolution integral:

\[
x_m(t) = \int_{-\infty}^{\infty} x(\tau)w(t-\tau)\,d\tau
\]

where \( w \) is a suitable weighting function.

Thus, the residual fluctuating part (rapidly-varying component) is defined as follows:

\[
x_f(t) = x(t) - x_m(t)
\]

The frequency-domain counterpart of Eq. (1) can be written as follows:

\[
\tilde{x}_m(n) = \hat{w}(n)\tilde{x}(n)
\]

where the hat symbol denotes the (generalized) Fourier transform operator and \( n \) is the frequency.

Eq. (3) shows that the kernel regression corresponds to the application of a filter to the original signal. The complex response function of the filter whose input is the original signal \( x(t) \) and whose output is the mean \( x_m(t) \) is provided by the Fourier transform of the weighting function \( \hat{w}(n) \).

The quantity \( |\hat{w}(n)|^2 \) is referred to as the transfer function of the slowly-varying mean part of the original signal.

Applying the Fourier transform also to Eq. (2), one obtains:

\[
\tilde{x}_f(n) = \tilde{x}(n) - \tilde{x}_m(n)
\]

Considering Eqs. (3) and (4), the Fourier transform of the fluctuating part is given by:

\[
\tilde{x}_f(n) = |1 - \hat{w}(n)|\tilde{x}(n)
\]

Analogously to Eq. (3), Eq. (5) shows that also the fluctuating component can be considered as the result of the application of a filter to the original signal. The complex response function of the filter whose input is the original signal \( x(t) \) and whose output is the fluctuating component \( x_f(t) \) is provided by the complement to unity of the Fourier transform of the weighting function \( 1 - |\hat{w}(n)| \). The quantity \( |1 - \hat{w}(n)|^2 \) is referred to as the transfer function of the residual fluctuating part of the original signal.

From Eqs. (3) and (5) it can be deduced that if for a specific frequency \( n \) both \( \hat{w}(n) \) and \( 1 - |\hat{w}(n)| \) are not null, the harmonic component of frequency \( n \) in the original signal is filtered but it is present both in the mean and in the fluctuating part. Thus, the mean and fluctuating parts are in general correlated. The product \( \hat{w}(n)|1 - \hat{w}(n)| \), where \( \bullet \) is the complex conjugate of \( \bullet \), is referred to as the joint-transfer function of the slowly-varying mean and residual fluctuating parts of the original signal; it provides a measure of the correlation between the mean and fluctuating components.
The linearity of Eqs. (1) and (2) gives rise to some noteworthy properties. In particular, when \( x(t) \) is a sample function of a Gaussian stationary random process \( X(t) \), then also the slowly-varying component and the rapidly varying component are Gaussian stationary random processes, denoted by \( X_m(t) \) and \( X_f(t) \), respectively. Their psdfs are given by:

\[
S_{X_m}(n) = \left| \hat{w}(n) \right|^2 S_X(n) \\
S_{X_f}(n) = \left| 1 - \hat{w}(n) \right|^2 S_X(n)
\]  

(6)

In general, \( X_m(t) \) and \( X_f(t) \) are incoherent only if their harmonic contents belong to different frequency ranges. If their frequency content is instead overlapping in some frequency range, they cannot be perfectly incoherent, and their cpsdf is given by:

\[
S_{X_mX_f}(n) = \hat{w}(n)\left| 1 - \hat{w}(n) \right|^2 S_X(n)
\]  

(7)

Eq. (7) shows that the cpsdf \( S_{X_mX_f}(n) \) is null for every frequency only if \( \hat{w}(n) \) is a step function: in that case, the product \( \hat{w}(n)\left| 1 - \hat{w}(n) \right|^2 \) is rigorously zero for every value of \( n \).

A number of different weighting functions can be adopted, whose Fourier transform can be defined analytically. Among these functions, the Constant weighting function (Section 2.1.1), normally adopted for the moving average extraction, the Gaussian weighting function (Section 2.1.2), also used by Su et al. (2015), and the cardinal Sine weighting function (Section 2.1.3), acting as a low-pass filter, are of particular interest for their peculiar properties and are considered here.

### 2.1.1. Constant weighting function

As it is usual, let us extract the slowly-varying mean component \( x_m(t) \) by Eq. (1), adopting the Constant weighting function (CW) given by:

\[
\hat{w}(n) = \begin{cases} 
\frac{1}{T} & |n| < \frac{T}{2} \\
0 & |n| > \frac{T}{2}
\end{cases}
\]  

(8)

The Fourier transform of \( w(t) \) is real and given by:

\[
\hat{w}(n) = \frac{\sin(\pi n T)}{\pi n T}
\]  

(9)

Fig. 1 plots the weighting function (Fig. 1a), together with its Fourier transform (Fig. 1b) and the complement to one of its Fourier transform (Fig. 1c). It points out that, as known in the literature (Smith, 2003), the moving average has little ability to separate one band of frequencies from another, since its transfer function in not uniformly decreasing with increasing the frequency. The Fourier transform of the CW has an oscillatory behaviour: it goes to zero at the frequency corresponding to the inverse of the window width \( 1/T \), but then it oscillates with decreasing amplitude.

Fig. 2 plots the three transfer functions \( |\hat{w}(n)|^2 \) (Fig. 2a), \( |1 - \hat{w}(n)|^2 \) (Fig. 2b), and \( \hat{w}(n)\left| 1 - \hat{w}(n) \right|^2 \) (Fig. 2c). Fig. 2a shows that the low-frequency components are retained in the slowly-varying mean part, but also higher frequency components are maintained, even if their amplitude is filtered. Fig. 2b shows that the fluctuating part preserves the high-frequency components of the original signal, amplifying the amplitude of some of them (the filter frequency response function is larger than one), and deamplifying some other. Fig. 2c shows that the joint-transfer function of the slowly-varying mean and fluctuating component is not null for some frequency bands, thus in general some correlation may exist between the mean and fluctuating component. The joint-transfer function has an oscillating behaviour involving positive or negative values. Such a behaviour gives rise to a noteworthy property. Although the joint transfer function is different from zero, its integral over the whole frequency range tends to be null. Accordingly, when dealing with a stationary random process whose psdf \( S_X(n) \) is nearly
white noise, the slowly varying mean and the residual fluctuation tend to be uncorrelated at the zero time lag.

### 2.1.2 Gaussian weighting function

As an alternative to the CW, the extraction of the slowly-varying mean by Eq. (1) may be performed adopting a Gaussian weighting function (GW) defined as:

\[
    w(t) = \begin{cases} 
    \exp\left(-\frac{\alpha^2 t^2}{2T^2}\right) & \text{if } |t| < \frac{T}{2} \\
    0 & \text{if } |t| > \frac{T}{2}
    \end{cases}
\]

(10)

If \( \alpha \) is greater than or equal to 12 (see Appendix A), then the effect of truncation of the window outside the interval \([-T/2, T/2]\) is negligible, the Fourier transform is real and it is given by:

\[
    \hat{w}(n) \simeq \exp\left(-\frac{\pi^2 n^2 T^2}{\alpha}\right)
\]

(11)

Fig. 3 plots the weighting function (Fig. 3a), together with its Fourier transform (Fig. 3b) and the complement to one of its Fourier transform (Fig. 3c) for different values of \( \alpha \geq 12 \). Differently from the CW, the adoption of a GW allows to filter out all the high frequency components from the mean. However, this operation does not separate clearly different harmonic components, since there is an intermediate range of frequencies which are maintained both in the mean part and in the fluctuating part. This is more evident for large values of the \( \alpha \) parameter (black lines in Fig. 3). Thus, if one wants to separate the harmonic content of the mean and fluctuating components, a small value of \( \alpha \) is suggested, provided that \( \alpha \geq 12 \).

### Fig. 3. Gaussian weighting function for three values of \( \alpha \) (light grey: \( \alpha = 12 \), dark grey: \( \alpha = 50 \), black: \( \alpha = 100 \)): (a) time window \( w(t) \), (b) Fourier transform of the time window \( \hat{w}(n) \) and its complement to unity \( 1 - \hat{w}(n) \) (c).

### Fig. 4. Gaussian weighting function for three values of \( \alpha \) (light grey: \( \alpha = 12 \), dark grey: \( \alpha = 50 \), black: \( \alpha = 100 \)): transfer functions.

### Fig. 5. Cardinal Sine weighting function: (a) time window \( w(t) \), (b) Fourier transform of the time window \( \hat{w}(n) \) and its complement to unity \( 1 - \hat{w}(n) \) (c).
2.1.3. Cardinal sine weighting function

The extraction of the slowly-varying mean by Eq. (1) may also be performed adopting a cardinal Sine weighting function (SW). It corresponds to a low-pass filter and is defined as:

\[ w(t) = \frac{\sin\left(\frac{\pi}{T} t\right)}{\pi} \]  

(12)

The Fourier transform of Eq. (12) is real and it is given by:

\[ \hat{w}(n) = \begin{cases} 1 & |n| < \frac{1}{T} \\ 0 & |n| > \frac{1}{T} \end{cases} \]  

(13)

Fig. 5 plots the weighting function (Fig. 5a), together with its Fourier transform (Fig. 5b) and the complement to one of its Fourier transform (Fig. 5c). The adoption of a SW allows to filter out all the high frequency components from the mean. Differently from the GW, the operation clearly separates different harmonic components. It should be remarked, however, that Eq. (13) holds if the window function in Eq. (12) is considered over an infinite support. The effect of the truncation of the window on a finite time \( T_n \), discussed in Appendix B, produces some fluctuations of the harmonic content in a neighborhood of the frequency \( n = 1/T \), whose amplitude should be carefully controlled.

Fig. 6 plots the three transfer functions \( |\hat{w}(n)|^2 \) (Fig. 6a), \( |1 - \hat{w}(n)|^2 \) (Fig. 6b), and \( \hat{w}(n)|1 - \hat{w}(n)| \) (Fig. 6c). Fig. 6a and (b) show that the SW acts as a low-pass filter: the mean component retains only the frequencies \( n < 1/T \), the fluctuating component preserves only the frequencies \( n > 1/T \). Fig. 6c shows that the joint-transfer function is null for every frequency. This means that when a stationary random process is considered, the mean and fluctuating components obtained adopting a CW are incoherent.

2.2. Reduced fluctuating component

Using the definition provided by Eqs. (1) and (2), the slowly-varying standard deviation of the fluctuating component can be defined as follows:

\[ \sigma_f(t) = \sqrt{\int_{-\infty}^{\infty} x_f^2(\tau)w(t-\tau)\,d\tau - \left[ \int_{-\infty}^{\infty} x_f(\tau)w(t-\tau)\,d\tau \right]^2} \]  

(14)

The reduced fluctuating component is then defined as:

\[ \tilde{x}_f(t) = \frac{x_f(t)}{\sigma_f(t)} \]  

(15)

It should be remarked that the signal decomposition into a mean and a fluctuating component (Eq. (1) and (2)) and the definition of the reduced fluctuating component (Eq. (14) and (15)) are usually combined and applied in sequence for thunderstorm velocity records (Chen and...
Letchford, 2004) and more recently also for synoptic velocity records (Zhang et al., 2018b, 2019). These two operations, however, are deeply different from each other: the moving average extraction involves the linear separation of different frequency bands, while the reduced fluctuation extraction involves a non-linear demodulation of the signal. Accordingly, even if \( x(t) \) is a Gaussian signal, in principle \( \tilde{x}_f(t) \) cannot retain this property. A wide literature proves that the reduced fluctuating component extracted from both synoptic Gaussian velocity records and thunderstorm non-Gaussian velocity records can be dealt with as Gaussian with a reasonable approximation (Zhang et al., 2018b, 2019). This topic will be further discussed in the following, highlighting some remarks about this traditional assumption.

3. Stationary winds

Synoptic winds are commonly schematized as a stationary random process, which can be decomposed as the sum of a constant mean wind velocity and three turbulence components (longitudinal, lateral and vertical), modelled as stationary Gaussian random processes. In this Section, the model adopted for the psdf of the three turbulence components is first introduced (Section 3.1). The consequences of the moving average extraction are analysed analytically in Section 3.2 considering three different time intervals \( T \). Finally, in order to verify numerically the analytical predictions on a representative set of sample functions, numerical simulations of the three turbulence components are carried out, the moving average extraction is performed numerically and the results are compared with analytical results in Section 3.3.

### 3.1. Power spectral density function of turbulence components

In the following analyses, the psdf of the three turbulence components is defined as (Solari & Piccardo, 2001):

\[
\frac{n_{S_u}(n)}{\sigma_u^2} = \frac{6.568 \frac{\sigma_u}{\mu}}{1 + (10.302 \frac{\sigma_u}{\mu})^{2/3}};n_{S_v}(n) = \frac{9.434 \frac{\sigma_v}{\mu}}{1 + (14.181 \frac{\sigma_v}{\mu})^{2/3}}; \frac{n_{S_w}(n)}{\sigma_w^2} = \frac{6.003 \frac{\sigma_w}{\mu}}{1 + 63.181 \left(\frac{\sigma_w}{\mu}\right)^{2/3}}
\]

(16)

### Table 1

<table>
<thead>
<tr>
<th>Component</th>
<th>( \sigma ) (m/s)</th>
<th>( L ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u )</td>
<td>6.16</td>
<td>48.25</td>
</tr>
<tr>
<td>( v )</td>
<td>4.62</td>
<td>12.06</td>
</tr>
<tr>
<td>( w )</td>
<td>3.08</td>
<td>4.82</td>
</tr>
</tbody>
</table>

**Fig. 8.** Longitudinal turbulence: theoretical evaluation of the psdf of the mean component, of the fluctuating component and of the cpsdf of the two components. Constant weighting function (top figures), Gaussian weighting function (middle figures), Cardinal Sine weighting function (bottom figures).
where \( u, v, w \) indicate the longitudinal, lateral, and vertical turbulence components, respectively; \( U \) is the mean wind velocity, \( \sigma \) and \( L_{\varepsilon} (\varepsilon = u, v, w) \) are the standard deviation and the integral length scale of \( u, v, w \). Fig. 7 plots the psdf of the three turbulence components as a function of the reduced frequency \( nL_{\varepsilon}/U \).

In the following evaluations, \( U = 21.6 \) m/s; Table 1 reports the values of the parameters in Eq. (16), evaluated according to CNR-DT 207 (2008) for a site in Exposition Category IV at a height \( z = 10 \) m above the ground.

### 3.2. Analytical evaluations

Analytical evaluations are carried out adopting three time intervals: \( T = 10 \) s, \( T = 30 \) s, \( T = 60 \) s, using a CW, a GW and a SW. When the GW is adopted, according to the discussion in Section 2, \( \alpha = 12 \) is assumed. In Figs. 8–10 the results obtained adopting the three time intervals and the three weighting functions are compared: light grey lines correspond to \( T = 10 \) s, grey lines to \( T = 30 \) s, black lines to \( T = 60 \) s; top Figures refer to the CW, mid Figures to the GW, bottom Figures to the SW.

Fig. 8 plots the psdf of the slowly-varying mean and fluctuating parts of the longitudinal turbulence, together with their cpsdf. It shows that adopting a small averaging time (light grey lines), the mean component retains a wide harmonic content and the areas under the psdf of the mean and fluctuating components are comparable. Furthermore, when a CW or a GW is adopted, the cpsdf is non negligible in an intermediate frequency range. On the contrary, when a large averaging time is adopted, the harmonic content of the mean component is mainly restricted to the low-frequency range and the areas under the psdf of the mean component is much smaller than the area under the psdf of the fluctuating component. Furthermore, both the CW and the GW do not allow a definite separation of the harmonic content between the mean and fluctuating components and the two components have non-negligible cpsdf. However, when a CW is adopted, the cpsdf has an oscillating behaviour and the area under the curve may be negligible, thus the mean and fluctuating components are almost uncorrelated. On the contrary, when a GW is adopted, the area under the cpsdf is always non null, and the two components are correlated. The SW allows a clear separation of the harmonic content of the mean and fluctuating components, and their cpsdf is rigorously null in the whole frequency range.

Fig. 9 plots the psdf of the slowly-varying mean and fluctuating parts of the lateral turbulence, together with their cpsdf. Results are similar to the ones obtained with reference to the longitudinal turbulence. However, since the harmonic content of the vertical turbulence is much more shifted towards the high frequency range, the psdf of the mean component, of the fluctuating component and of the cpsdf of the two components. Constant weighting function (top figures), Gaussian weighting function (middle figures), Cardinal Sine weighting function (bottom figures).
component is negligible with respect to the psdf of the fluctuating component, which is comparable to the original psdf in Fig. 7, especially when a large averaging time is considered \( T = 60 \) s. Furthermore, the cpsdf of the mean and fluctuating components is much smaller than the one obtained for the longitudinal and lateral turbulence, and it is also almost negligible when a CW or a GW are adopted.

As a final remark, the comparison between the results in Figs. 8–10 point out that the filtering effect associated with the moving mean extraction is related to the length of the window, but it is different for the three turbulence components, due to their different harmonic content.

3.3. Numerical simulations

In this Section, the results of Monte Carlo simulations are reported. The three turbulence components are simulated starting from the psdf in Section 2. The parameters in Table 1 are adopted (Solari and Piccardo, 2001, CNR-DT 207, 2008). According to the random phase method (Shinozuka and Jan 1972), turbulence time histories are generated as a series of cosine functions with weighted amplitudes, evenly spaced frequencies, and random phase angles. The sampling period is set \( T_s = 0.1 \) s, and 100 sample functions are simulated for each turbulence component. The slowly-varying mean wind velocity is evaluated adopting a CW, a GW (with \( \alpha = 12 \)) and a SW of length \( T = 30 \) s. Section 3.3.1 reports the results of the moving average extraction, comparing numerical psdfs with analytical predictions. Section 3.3.2 reports the numerical results dealing with reduced fluctuations.

3.3.1. Moving average extraction

Fig. 11 plots a sample function of the longitudinal turbulence, together with the slowly-varying mean component: Fig. 11a plots the complete time history \( (T_t = 600 \) s), Fig. 11b plots the same sample function on a shorter time window \( (60 \) s). Fig. 11a shows that the trend of the moving average is apparently the same adopting the three types of weighting functions. However, Fig. 11b shows that adopting a GW or a SW the slowly-varying mean follows more closely the low frequency variation of the time history.

Fig. 12 compares the numerically-evaluated psdf of the original, mean and fluctuating components with the analytical estimates in Eq. (6). It confirms the analytical prediction and shows that the moving average operation with a CW does not realize a filter at lower frequencies: the fluctuating part of the signal presents some amplified harmonic components with respect to the original one. Furthermore, it shows that only the SW is able to provide a separation of the harmonic content of the mean and fluctuating components. Adopting a CW or a GW, a frequency range exists where both the mean and the fluctuating components have non-negligible psdf; thus they are partially coherent. Taking into account the coherence between the two components may be essential in the analysis of the dynamic response of structures based on the
The wind velocity can be decomposed into a slowly-varying mean and a fluctuating component.

Figs. 13 and 14 are analogous to Figs. 11 and 12, but are related to the lateral turbulence component.

Figs. 15 and 16 are analogous to Figs. 11 and 12, but are related to the vertical turbulence component. Both the time history and the psdf show that the mean component is almost null and it has negligible psdf. Thus, the moving average operation has an almost negligible effect on the signal and the fluctuating component is almost coincident with the original signal itself.

Table 2 reports the standard deviation of the three turbulence components, of the mean and fluctuating parts and their correlation coefficient. Results obtained adopting a CW, a GW and a SW are reported. The extraction of the slowly-varying mean from a signal, filtering out some harmonic components, determines a reduced standard deviation. This is more evident when a GW is adopted. Furthermore, the correlation between the mean and fluctuating parts is null only when a SW is adopted. The CW and GW lead to a non-negligible correlation. The correlation is higher when a GW is adopted. For the vertical component, since the harmonic content is shifted towards the higher frequencies with respect to the longitudinal and lateral turbulence, the standard deviation of the mean component is smaller, as well as the correlation between the mean and fluctuating components. The correlation between the mean and fluctuating parts has to be appropriately taken into account in the analysis of the dynamic response of structures.

Fig. 17 compares the psdf of the three turbulence components and their fluctuating parts. It shows that the filtering effect associated with the extraction of the slowly-varying mean is remarkable for the longitudinal component and it becomes almost negligible for the vertical component.

As a final remark, it should be pointed out that a clear separation of the harmonic content of the mean and fluctuating components is not possible adopting a moving average operation with a CW or a GW. On the other hand, a clear separation can be achieved with a SW; however, the sharp separation of the harmonic content provided by the SW, makes the selection of the window length a very delicate point as well as the presence of some fluctuations of the harmonic content around the frequency $n = 1/T$ (Appendix B).
The overall results of these analyses demonstrate that each of the considered windows may have advantages and drawbacks. The CW has the drawback of distorting the harmonic content of both the slowly varying mean and fluctuating components; the GW does not allow a clear separation of the different harmonic contents; the adoption of a SW in principle solves the above shortcomings but involves a delicate choice of both the signal and window lengths.

Table 3 reports the integral length scale of the three turbulence components, together with the integral length scales of the fluctuating components. The integral length scale is estimated from the time instant at which the normalized auto-covariance is 1/e (Flay and Stevenson, 1988): the estimate based on the integral of the covariance function leads to larger and less reliable values.

It can be deduced that, as a consequence of the subtraction of the low-frequency components, the integral length scales of the longitudinal and lateral fluctuating components are smaller than the original ones. Furthermore, due to the fact that the fluctuating components have much more similar harmonic content, their integral length scales are closer, as already observed by Zhang et al. (2019). Dealing with the vertical turbulence component, since its harmonic content is shifted at higher frequencies, for all the three considered windows, the fluctuating part is almost coincident with the original time history and the integral length scale of the fluctuating component is the same and it coincides with the one of the original component.

3.3.2. Reduced turbulent fluctuations

This Section provides a numerical analysis of the effect of the extraction of the slowly-varying standard deviation for the definition of the reduced turbulent fluctuations. Fig. 18 plots the results obtained for the longitudinal turbulence. For the sake of brevity, analogous Figures related to the lateral and vertical components are not reported, since the results are totally in agreement with those reported for the longitudinal component. Fig. 18a plots a sample time history of the fluctuating component (Eq. (4)) obtained adopting the three different weighting functions studied in this paper, Fig. 18b plots the corresponding slowly-varying standard deviation (Eq. (14)), Fig. 18c shows...
Fig. 15. Sample function of the vertical turbulence component, together with its moving average: (a) 10 min, (b) 1 min.

Fig. 16. Psdf of the vertical turbulence, together with its mean and fluctuating components.

Fig. 17. Psdf of the three turbulence components, together with their fluctuating components.
the reduced turbulent fluctuation (Eq. (15)), and Fig. 18d compares the normalized psdf of the fluctuating components obtained with the three different weighting functions with the psdf of the corresponding reduced turbulent fluctuation. Fig. 18b shows that the slowly-varying standard deviations estimated adopting a GW or a SW are lower and much smoother than the one obtained adopting a CW. Fig. 18d shows that the psdf of the fluctuating component is almost unaffected by the demodulation with the moving standard deviation: the normalized psdf of the fluctuating component is almost coincident with the psdf of the reduced turbulent fluctuation. A comparison between the psdf of the reduced turbulence components extracted with the three different windows shows that the psdf of the reduced fluctuation extracted adopting a CW is characterized by unphysical oscillations in the low-frequency range, associated to the shape of its transfer function (Fig. 2). Some less evident oscillations are detectable also in the psdf of the reduced fluctuation extracted through the SW, an issue that is probably attributable to the presenting a large number of outliers are rejected, and 13 thunderstorms, considered as the most reliable, are taken into account for this analysis.

Section 4.1 recalls the fundamentals of the directional decomposition strategy proposed by Zhang et al. (2019). Section 4.2 applies this method in order to determine the three wind velocity components of the selected thunderstorm records. In this framework, the slowly-varying mean wind velocity is extracted using a CW, a GW and a SW and the psdf of the three reduced turbulence components are analyzed.

4.1. Directional decomposition

Let $V_X$, $V_Y$, $V_Z$ be the three components of the wind velocity recorded by an anemometer in the geophysical coordinate system, with $V_X$ directed from West to East, $V_Y$ from South to North and $V_Z$ vertical and positive upwards. Let us express the three Cartesian components of the velocity as follows:

$$V_X(t) = \bar{V}_X(t) + \epsilon_X(t)$$
$$V_Y(t) = \bar{V}_Y(t) + \epsilon_Y(t)$$
$$V_Z(t) = \bar{V}_Z(t) + \epsilon_Z(t)$$

where $\bar{V}_X(t)$, $\bar{V}_Y(t)$ and $\bar{V}_Z(t)$ are the slowly-varying mean velocity components; they can be extracted adopting different weighting functions. The horizontal-plane component of the slowly-varying mean wind velocity and its direction are given by:

$$\pi(t) = \sqrt{\bar{V}_X^2(t) + \bar{V}_Y^2(t)}$$
$$\pi(t) = 270 - \arctan(\frac{\bar{V}_Y(t)}{\bar{V}_X(t)})$$

$$\bar{\beta}(t) = 270 - \pi(t)$$

where $\alpha \in [0360^\circ]$ is a clockwise directional angle, computed from the North, $\beta \in [0360^\circ]$ is an anti-clockwise directional angle, computed from the East (Fig. 19). Accordingly, $V_Y$ and $V_Z$ are projected onto a new Cartesian reference system with the $x$-axis coincident with the direction of $\bar{\beta} = \pi(t)$ and rotated $\bar{\beta} - \bar{\beta}$ with respect to the West-East direction. Thus, the turbulence components are given by:

$$u(t) = V_Y(t)\cos(\bar{\beta}(t)) + V_Z(t)\sin(\bar{\beta}(t))$$
$$v(t) = -V_Y(t)\sin(\bar{\beta}(t)) + V_Z(t)\cos(\bar{\beta}(t))$$
$$w(t) = V_X(t)$$

The turbulence components can also be expressed as follows:

Since the objective of this paper is the analysis of the three Cartesian wind velocity components, only the records provided by three-axial anemometers are considered. In particular, the thunderstorm outflows recorded by the anemometer providing the largest number of reliable records (referred to as Anemometer 1 in Solari et al., 2012), located in the Port of Livorno, are processed. It is an ultrasonic tri-axial anemometer located on a tower at a height $z = 20$ m above the ground, data are collected with a sampling rate 10 Hz, wind measurements are collected with a precision of 0.01 m/s and 1° for wind speed and direction, respectively. After an accurate analysis of the recorded time histories previously classified as thunderstorms (Zhang et al., 2018b), the ones presenting a large number of outliers are rejected, and 13 thunderstorms, considered as the most reliable, are taken into account for this analysis.

Table 2

<table>
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<th></th>
<th>CW</th>
<th>GW</th>
<th>SW</th>
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<td>$\rho$</td>
<td>$\sigma_{\epsilon}$</td>
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Table 3

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<th>$l_{cv}$ (m)</th>
<th>$l_{cf}$ (m)</th>
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<td>28.1</td>
<td>19.4</td>
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<td>$\epsilon$</td>
<td>17.3</td>
<td>13</td>
<td>10.8</td>
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<tr>
<td>$\epsilon$</td>
<td>6.48</td>
<td>6.48</td>
<td>6.48</td>
</tr>
</tbody>
</table>

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Section 4.1 recalls the fundamentals of the directional decomposition strategy proposed by Zhang et al. (2019). Section 4.2 applies this method in order to determine the three wind velocity components of the selected thunderstorm records. In this framework, the slowly-varying mean wind velocity is extracted using a CW, a GW and a SW and the psdf of the three reduced turbulence components are analysed.

4.1. Directional decomposition

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where $\bar{V}_X(t)$, $\bar{V}_Y(t)$ and $\bar{V}_Z(t)$ are the slowly-varying mean velocity components; they can be extracted adopting different weighting functions. The horizontal-plane component of the slowly-varying mean wind velocity and its direction are given by:

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where $\alpha \in [0360^\circ]$ is a clockwise directional angle, computed from the North, $\beta \in [0360^\circ]$ is an anti-clockwise directional angle, computed from the East (Fig. 19). Accordingly, $V_Y$ and $V_Z$ are projected onto a new Cartesian reference system with the $x$-axis coincident with the direction of $\bar{\beta} = \pi(t)$ and rotated $\bar{\beta} - \bar{\beta}$ with respect to the West-East direction. Thus, the turbulence components are given by:

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$$v(t) = -V_Y(t)\sin(\bar{\beta}(t)) + V_Z(t)\cos(\bar{\beta}(t))$$
$$w(t) = V_X(t)$$

The turbulence components can also be expressed as follows:
\[ u'(t) = \sigma_u(t) \tilde{u}(t) \text{ and } \sigma_u(t) = \sigma_u(t) \tilde{u}'(t) \]

where \( \sigma_u(t) \), \( \sigma_v(t) \), \( \sigma_w(t) \) are the slowly-varying standard deviations, \( \tilde{u}(t) \), \( \tilde{v}(t) \), \( \tilde{w}(t) \) are the reduced turbulence components. Zhang et al. (2019) modelled them as uncorrelated Gaussian random processes.

**4.2. Data analysis**

Fig. 20 plots the resultant horizontal wind velocity together with its slowly-varying mean (Fig. 20a and b), its direction together with its slowly-varying mean (Fig. 20c), and the vertical velocity component with its slowly-varying mean (Fig. 20d) for a thunderstorm event. CW, GW and SW are adopted, and the window length \( T = 30 \) s is used. As already
observed for the numerically-simulated stationary turbulence components, the trend of the slowly-varying mean velocity is similar for the three weighting functions. However, Fig. 20b shows that adopting a GW or a SW the slowly-varying mean follows more closely the low frequency variation of the time history. This different behaviour is related to the different shape of the Fourier transform of the CW, GW and SW (Figs. 1, 3) variation of the time history. This different behaviour is related to the different shape of the Fourier transform of the CW, GW and SW (Figs. 1, 3) and it has relevant consequences on the psdf of the turbulence components, as discussed in Fig. 24.

Fig. 21 plots the three residual fluctuating components obtained through the three weighting functions: their time histories are non-stationary and the results obtained adopting the three weighting functions are comparable.

Fig. 22 plots the slowly-varying standard deviation of the three turbulence components. It shows that the standard deviation of the longitudinal turbulence is higher than the other two components at some time ranges, while the standard deviation of the three turbulence components are comparable in other ranges of time. Furthermore, as already observed with reference to stationary events (Table 2), the comparison between CW, GW and SW shows that adopting GW and SW lower standard deviations are obtained for the turbulence components. This is a consequence of the different filtering associated to the three weighting functions considered (Section 2). The time evolution of the standard deviations of the three turbulence components are comparable.

Fig. 23 plots the time history of the three reduced turbulence fluctuations. A test for stationarity has been performed on all the reduced turbulence components (Bendat and Piersol, 2012). The time histories have been divided into 20 equal time intervals of length 30 s, the mean square value has been computed for each interval and the reverse arrangements test has been applied, verifying stationarity at 5% level of significance. As already observed by Solari et al., 2015a, Zhang et al. (2018b, 2019) and many other authors, it was confirmed that the demodulation of the turbulence components though Eq. (15) makes the reduced components stationary.

Table 5 reports the mean values of the skewness and kurtosis of the reduced turbulence components, estimated from 13 records. As already observed for the simulated synoptic events, the reduced turbulent fluctuations are slightly non-Gaussian especially with regard to the kurtosis values. This feature, already noted by Solari et al., 2015a and Zhang et al. (2018b, 2019) without providing any interpretation, may be ascribed to the nonlinear demodulation of the signal associated with the reduced fluctuation definition. It should be remarked that the deviation of the reduced fluctuation from the Gaussian distribution is more apparent when the SW is adopted.

Fig. 24 plots the average psdf of the three reduced turbulence components, estimated from 13 thunderstorm records, comparing the results obtained adopting CW, GW and SW and the analytical expressions in Eq. (16). It confirms that, as already observed with reference to synoptic events (Figs. 12, 14 and 16), the largest differences among different weighting functions are obtained when the longitudinal turbulence is considered. On the other hand, the choice of different weighting functions has almost negligible effects on the harmonic content of the vertical reduced turbulence.

When the longitudinal reduced fluctuation is considered, the adoption of a CW leads to an unphysical oscillating behaviour in the low frequency range of the psdf. This result had already been obtained in previous investigations (Solari et al., 2015a; Zhang et al., 2018b, 2019). Initially (Solari et al., 2015a), the low frequency peak was potentially ascribed to an inherent property of thunderstorms; later on Zhang et al. (2018b, 2019) associated this peak to the difficulty of separating the low and high frequency ranges without the spectral gap. The analytical and numerical considerations carried out in this paper definitely prove that the low-frequency peak is related to the oscillating shape of the transfer function associated to the GW (Fig. 2). Similarly, as already observed for synoptic events (Fig. 18), also adopting a SW, some oscillations persist and are detectable in the psdf of the longitudinal reduced fluctuation. Also in this case such oscillations may be related to the oscillating trend of the transfer function for the finite length window highlighted in Appendix B. However, their effect seems to be negligible in the analysis of the reduced turbulence extracted from stationary events, while it seems to be amplified for non-stationary thunderstorm records. Diversely, when a GW is applied, the reduced turbulence has a smoother psdf, comparable with the one typical of synoptic winds, filtered in the low frequency range due to the moving average operation. Thus, the GW is the most reliable in preventing from unphysical oscillations the harmonic content of the reduced turbulence. It should be in any case remarked that the less regular behaviour of the estimated psdf with respect to the ones in Fig. 18 may be ascribed to the much smaller number of samples adopted for the psdf estimate (13 samples, with respect to 100 samples adopted in the numerical analyses of synoptic events). The availability of a larger reliable database of thunderstorm outflow records could put the basis for the formulation of an analytical reliable spectral model of the reduced turbulence. Results here presented with reference to a limited database show that the model for the psdf could be of the same type adopted for synoptic winds (Eq. (16)), filtered in the low frequency range according to Eq. (6), depending on the weighting function adopted for the slowly-varying mean component extraction. Finally, it should be recalled that the width of the frequency band filtered by the moving average extraction is a function of the window width (Sections 2, 3.2). Results plotted in Fig. 24 refer to a window width $T = 30$ s. Appendix C reports results obtained with different window widths: adopting a smaller window width (e.g. $T = 10$ s), the filter in the low frequency range associated with the moving average extraction is extended to a wider frequency range, while adopting a wider window (e.g. $T = 60$ s) the filtering effect is limited to very low frequencies.

<table>
<thead>
<tr>
<th>$\epsilon = \bar{u} u$</th>
<th>$\epsilon = \bar{v} v$</th>
<th>$\epsilon = \bar{w} w$</th>
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<tr>
<td>kurt, (GW)</td>
<td>2.7</td>
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</tr>
<tr>
<td>kurt, (SW)</td>
<td>2.9</td>
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Table 4 Kurtosis of the reduced fluctuating turbulence components.
5. Conclusions

In this paper, moving average operations are analysed considering different weighting functions, namely a Constant Weight (CW), a Gaussian Weight (GW) and a Cardinal Sine Weight (SW). The moving average operation is interpreted as a filter, whose transfer function is evaluated analytically. The filter effect associated to the moving average operation depends on the time length of the window and on the shape of the weighting function. The advantages and disadvantages of adopting a CW, a GW or a SW are discussed. With respect to the CW, commonly adopted in moving average operations, the GW allows a better separation of the harmonic content between the mean and the fluctuating...
components; furthermore, it involves a smooth transfer function which does not give rise to unphysical peaks in the psdf of the fluctuating component. On the contrary, the CW is characterized by oscillations in the transfer function which create a distortion of the harmonic content. However, adopting a CW reduces the correlation between the mean and fluctuating components. The SW allows a clear separation of the harmonic content between the mean and fluctuating components, which are incoherent and uncorrelated; however, it involves a delicate choice of the window length and does not preserve the harmonic content of the fluctuating component from some unphysical oscillations in the low frequency range.

When dealing with synoptic winds, the three turbulence components are modelled as stationary random processes, and the psdfs of the slowly-varying mean and fluctuating component as well as their cpsdf are predicted analytically. Fixing a weighting function and a time length of the window, due to their different harmonic content, the filtering effect associated with the moving average extraction is different for the three turbulence components, e.g. it is almost negligible when the moving average is extracted with a time length of the window $T = 30 \text{ s}$ on the vertical turbulence component. Under this point of view it could be reasonable to consider different time lengths of the window for different turbulence components.

The analytical and numerical evaluations show that when subtracting from a wind velocity time history its moving average, the residual fluctuations have smaller variance and integral length scale. Furthermore, the effect of such subtraction is much more evident when dealing with

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**Fig. 21.** Sample thunderstorm turbulence components: longitudinal (a), lateral (b), vertical (c).
the longitudinal component of the turbulence, which has a lower frequency harmonic content. This aspect is essential to carry out appropriate comparisons between synoptic wind speeds and thunderstorm outflows.

The definition of the reduced fluctuating components, commonly applied for thunderstorm velocity records, involves a nonlinear demodulation of the original velocity signal and cannot be interpreted within the theory of the linear transformations of random processes. Also when considering stationary Gaussian turbulence fluctuations, their

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**Fig. 22.** Sample thunderstorm: moving standard deviation of the three turbulent components.

**Fig. 23.** Sample thunderstorm reduced turbulent components: longitudinal (a), lateral (b), vertical (c).
demodulation to define their reduced counterparts leads to non-Gaussian random processes. Non-Gaussian features, however, are slight and mostly related to the kurtosis value.

The application of different weighting functions to non-stationary thunderstorm records confirms the analytical predictions formulated for stationary synoptic winds. A particular feature of thunderstorm velocity records that was not fully understood in previous research, namely the presence of low-frequency peaks in the psdf of the reduced turbulence components, is here interpreted as a consequence of the filtering effect associated to the moving average extraction with a CW. The adoption of a SW, that in principle should provide a clear separation of the harmonic content of the mean and fluctuating components, gives rise to some oscillations in the harmonic content of the reduced turbulence which could be probably ascribed to the finite length of the window. The GW provides reduced turbulence components with psdfs comparable with the models adopted for synoptic winds, but filtered in the low-frequency range.

As far as concerns the pdf of the reduced turbulent fluctuation, its weakly non-Gaussian nature can be interpreted not as a specific feature of the thunderstorm outflows but rather as a consequence, clearly apparent also for synoptic wind speeds, of the nonlinear demodulation of the original signal.

The systematic application of these procedures to a large thunderstorm outflow database could put the basis for the formulation of an analytical reliable spectral model of the reduced turbulence, essential for the assessment of the dynamic response of structures to downbursts.

Furthermore, the choice of the weighting function with regard to the correlation between the slowly varying mean and the fluctuating components, could be re-evaluated and interpreted with reference to its consequences in the evaluation of the dynamic response of structures. The stationary characteristics of the reduced turbulence fluctuation

| Table 5 |

| Thunderstorm records: skewness and kurtosis of the reduced fluctuating turbulence components. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| skew_u (CW)     | skew_u (GW)     | skew_u (SW)     | kurt_u (CW)     | kurt_u (GW)     | kurt_u (SW)     |
| skew_v (CW)     | skew_v (GW)     | skew_v (SW)     | kurt_v (CW)     | kurt_v (GW)     | kurt_v (SW)     |
| skew_w (CW)     | skew_w (GW)     | skew_w (SW)     | kurt_w (CW)     | kurt_w (GW)     | kurt_w (SW)     |
| skew_u          | skew_v          | skew_w          | kurt_u          | kurt_v          | kurt_w          |

Fig. 24. Thunderstorm psdf of the three reduced turbulent components: longitudinal (a), lateral (b), vertical (c).
supports the possibility of schematizing thunderstorms as modulated stationary random processes. However, a systematic comparison of kernel regression techniques and advanced procedures for the moving mean extraction, such as wavelet transforms, is highly advisable.

Declaration of competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

**Federica Tubino:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data curation, Writing - original draft, Writing - review & editing, Visualization. **Giovanni Solari:** Conceptualization, Investigation, Resources, Writing - review & editing, Funding acquisition, Supervision.

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Appendix A

In this Appendix, GW (Eq. (10)) with different values of the $\alpha$ parameter are considered in order to check the validity of Eq. (11) in the estimation of the Fourier transform. Figure A1 plots the weighting function (Fig A1a) and the absolute value of its Fourier transform, which is generally a complex function (Fig A1b). In Fig. A1b, the Fourier transform estimated numerically (dots) is compared with the analytical expression in Eq. (11). It can be deduced that for very small values of $\alpha$ (e.g. $\alpha = 1$), the bell shape of the Gaussian function is truncated in the time domain, and the weighting function is almost constant. In these cases, Eq. (11) does not provide a reasonable approximation of the Fourier transform of the weighting function, which has an oscillatory behaviour similar to the one typical of a CW. For greater values of $\alpha$, the Gaussian bell is not truncated and the Fourier transform is well provided by the analytical expression in Eq. (11). The limit value of $\alpha$ for avoiding the oscillatory behaviour of the Fourier transform may be obtained by setting $w(t)=\epsilon w(0)$ with $\epsilon$ small enough; setting $\epsilon = 0.05$, $\alpha = 12$ is obtained.

![Fig. A1. Gaussian weighting function: time domain (a) and comparison between numerical (dots) and analytical (lines) Fourier transform (b).](image)

Appendix B

In this Appendix, the effect on the Fourier Transform of the truncation of the SW on a finite length $T_f$ is studied numerically. Fig. B1 compares the theoretical Fourier transform of the window (Eq. (13)) with the discrete Fourier transform estimated for a window of finite length $T_f$. This figure shows that the finite length of the window does not allow the perfect low-pass filtering of the signal since the discrete Fourier transform of the finite length window is not a perfect unit box function. The truncation effect is almost negligible in the low-frequency range if the length of the window is large enough compared with $T$, but some oscillations of the Fourier transform of the window are always detectable around the frequency $n = 1/T$. 

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Appendix C

In this Appendix, the psdf of the reduced turbulent components for thunderstorm outflows obtained from analyses adopting \( T = 10 \, \text{s} \) and \( T = 60 \, \text{s} \) are reported. Analogously to Fig. 24, they confirm that adopting a CW or a SW unphysical oscillations in the reduced turbulence psdf are obtained. Furthermore, as discussed in Section 3.2 with reference to stationary winds, adopting \( T = 10 \, \text{s} \), the filter in the low frequency range associated with the moving average extraction is extended to a wider frequency range, while adopting \( T = 60 \, \text{s} \) the filtering effect is limited to very low frequencies. The adoption of different time intervals for the definition of the slowly-varying mean component has repercussions on the estimate of the dynamic response of structures. If a small time interval (e.g. \( T = 10 \, \text{s} \)) is adopted, a wider harmonic content is retained in the mean component: in this case the use of the statics for the estimate of the structural response to the slowly-varying mean aerodynamic loading could not be reliable. The evaluation of the response to the mean aerodynamic loading could require a dynamic analysis.

Fig. B1. Cardinal Sine weighting function: comparison between numerical (dots) and analytical (lines) Fourier transform (a) \( T_T = 20 \, T \), (b) \( T_T = 200 \, T \).
Fig. C1. Thunderstorm psdf of the three reduced turbulent components: longitudinal (a), lateral (b), vertical (c) ($T = 10$ s).
Fig. C2. Thunderstorm psdf of the three reduced turbulent components: longitudinal (a), lateral (b), vertical (c) ($T = 60$ s).

References


